

USING A DIRECT SEARCH ALGORITHM TO OPTIMIZE SPECIES COMPOSITION IN UNEVEN-AGED FOREST STANDS

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ABSTRACT. Describes an approach for determining the optimal sustainable equilibrium diameter distribution and species composition for uneven-aged forest stands. A direct search, derivative free, constrained nonlinear programming algorithm is applied to a deterministic version of the Stand Prognosis Model. The diameter distribution for each species is described by a two-parameter Weibull distribution and the number of trees per acre. The optimization problem is formulated in terms of these three decision variables per species. Results are presented for both board and cubic foot growth objective functions, and the species composition is allowed to consist of one to three species. Few of the optimal solutions produce balanced diameter distributions, although all are sustainable over the cutting cycle. Solutions involving a mixture of the three permissible species produce more volume growth than do either the one or two species alternatives.

INTRODUCTION

Determining the optimal sustainable equilibrium diameter distribution for an uneven-aged mixed-species forest stand is one of the major decisions facing forest managers (Hann and Bare, 1979). Yet, only recently have forest researchers begun to address these problems (e.g., Bare and Opalach, 1987; Haight, 1985, 1987; Hansen and Nyland, 1987; Martin, 1982; Adams and Ek, 1974). Using a deterministic version of the Stand Prognosis Model, a distance independent individual tree growth model with species-dependent growth dynamics, this paper discusses a procedure for determining the optimal sustainable equilibrium diameter distribution and species composition for uneven-aged forest stands. For comparative purposes, optimal solutions are derived for both board and cubic foot growth objectives consisting of one to three species.

MODEL DEVELOPMENT

The deterministic version of the Stand Prognosis Model developed for purposes of optimization consists of three parts: (1) large and small tree diameter increment functions, (2) mortality functions, and (3) regeneration functions. Diameter increment functions are taken from Wykoff (1986), but random perturbations are excluded. Mortality functions are taken from Wykoff et al. (1982) and Wykoff (1986). In the Prognosis Model, trees are removed from the tree list if their probability of survival compares unfavorably with a randomly drawn number. However, the deterministic version of the Prognosis Model keeps

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track of the number of trees per acre represented by each tree on the list. At the end of each ten-year projection period, the number of trees per acre represented by each tree on the list is updated by multiplying its "survival tree factor" by the computed probability of survival. This method for assigning mortality is very similar to the deterministic mortality option of the STEMS growth projection system (Belcher et al., 1982).

Regeneration functions are derived from Ferguson et al. (1986) and the FORTRAN source code for version 5.0 of the Prognosis Model. To implement regeneration functions in a deterministic manner, probabilities are viewed as proportions. For example, Ferguson et al. (1986) assign the three classes of regeneration (i.e., advance, subsequent, and excess) for each species to 1/300-acre plots by comparing random numbers to computed probabilities. In the deterministic version of the Prognosis Model, these probabilities are viewed as proportions and are multiplied by 300 to determine the number of advance, subsequent, and excess trees of each species to add to the tree list. In doing this, it is assumed that each 1/300-acre plot can have at most one advance, one subsequent, and one excess regeneration tree of a given species.

In addition to the assumptions outlined above, it is also assumed that best trees (i.e., advance and subsequent regeneration) are at least 4.5 feet in height. Best trees are a crucial component of a residual stand and are added to the tree list to determine sustainable equilibrium diameter distributions.

Given a beginning tree list (made up of one or more species), the deterministic version of the Prognosis Model first uses ten-year diameter increment functions to update tree diameters. Mortality functions are then used to compute ten-year survival proportions. Lastly, trees are added to the tree list by the regeneration subsystem and volumes are computed to evaluate the objective function. This sequence is repeated as many times as necessary to account for the cutting cycle being evaluated.

The sustainable equilibrium diameter distribution for each species is modeled by a two-parameter Weibull distribution function. Thus, the optimization problem is formulated in terms of three decision variables per species: (1) the scale and shape parameters of the Weibull distribution, and (2) the total number of trees per acre. As described later, a tree list is constructed for each set of these decision variables generated during the optimization process.

Box's complex algorithm (Kuester and Mize, 1973) is used to solve the optimization problem. The first step in the optimization process is to construct an initial complex. This is a set of points in decision space, which consists of $3 \times K$ dimensions, where K is the number of species being considered for the residual stand. Associated with each point in the complex is an objective function value. Initially, the points in the complex are generated to ensure broad coverage of the decision space, and for each point in the initial complex, a tree list is constructed. This tree list is passed to the deterministic version of the Prognosis Model and an updated tree list is returned representing the status of the stand

at the end of the cutting cycle. After evaluating the objective function, the equilibrium sustainability constraint is checked and the objective function value is penalized if the constraint is violated.

Twenty-one solutions are used to construct the initial complex for the three species optimization problem. The algorithm then begins an iterative search to locate the optimal solution. The point in the complex with the lowest objective function value is omitted and the centroid of the remaining points is computed. The lowest-valued point and the centroid are then used to define the search direction, and a new solution is located in this direction. This new solution is used to generate a new tree list and the deterministic version of the Prognosis Model is used to update the list and provide an associated value of the objective function. This sequence continues until convergence criteria are satisfied or the number of iterations exceeds a prespecified limit.

MATHEMATICAL MODEL

The general form of the optimization model described in the previous section is:

$$\text{MAX } Z_t = \sum_{s=1}^K \sum_{u=1}^{N'_s} V_{su} R_{su} - \sum_{s=1}^K \sum_{u=1}^{N_s} V_{su} R_{su}$$

Subject to:

$$X'_{ds} - X_{ds} \geq 0 \text{ for } d = 1, 2, \dots, M \\ s = 1, 2, \dots, K$$

$$B_s > 0, \quad C_s > 0, \text{ and } N_s \geq 0 \text{ for } s = 1, 2, \dots, K$$

Where,

Z_t = Per acre board (cubic) foot volume growth harvested every t -years

t = Cutting cycle ($t = 10$ NGP)

NGP = Number of ten-year growth projection periods in cutting cycle

K = Number of species in residual stand

M = Number of diameter classes used in equilibrium sustainability constraint

N_s, N'_s = Number of trees per acre of s th species at beginning and end of cutting cycle

V_{su} = Scribner board foot or cubic foot volume for u th tree, s th species

R_{su} = Survival tree factor. The number of trees per acre represented by uth tree, sth species

X_{ds}, X'_{ds} = Number of trees per acre in dth diameter class for sth species at beginning and end of cutting cycle

B_s = Weibull distribution scale parameter for sth species

C_s = Weibull distribution shape parameter for sth species

The relationship between X_{ds} and the three decision variables (B_s , C_s , and N_s) is shown in the following equation (Martin, 1982; Bailey and Dell, 1973):

$$X_{ds} = \frac{N_s \{ \exp [-(DL_{ds}/B_s)^{C_s}] - \exp [-(DU_{ds}/B_s)^{C_s}] \}}{1 - \exp [-(MD/B_s)^{C_s}]}$$

In this formula, MD, DL_{ds} , and DU_{ds} are, respectively, the diameter of the largest tree permitted in the residual diameter distribution, and the lower and upper diameter limits for the dth diameter class and sth species. For all results presented here, the width of the diameter class used in the sustainability equilibrium constraint is three inches and the maximum tree allowed in the residual stand is 27 inches. This equation is used to generate each beginning tree list by assuming that trees are uniformly distributed within each diameter class. This produces initial distributions which are approximately Weibull distributed.

RESULTS

To demonstrate the use of the model, the Abies lasiocarpa/Clintonia uniflora habitat type on the Coeur d'Alene National Forest in northern Idaho is highlighted. Species commonly found in this type include Abies lasiocarpa, Picea engelmannii, Pseudotsuga menziesii, Larix occidentalis, Pinus contorta, and Pinus monticola. Although this common forest type occurs under a wide variety of growing conditions, all results presented below assume an elevation of 4,500 feet, a slope of ten percent, and an aspect of zero degrees. Uneven-aged silvicultural systems are used in this type to favor watershed, recreation, wildlife, and amenity values (Alexander and Edminster, 1977).

At most three species--limited to the first three species in the above species list--are permitted in the residual stand. Other species entering the stand during the cutting cycle are assumed to be removed in a precommercial thinning operation during the harvest at the end of the cycle. Board foot volumes are only computed for trees ≥ 9 inches in diameter and cubic foot volumes are computed for all trees.

Analyses were conducted using a ten-year cutting cycle for both the board and cubic foot growth objectives. Tables 1 and 2 contain the steady-state diameter distributions for the three species mixture for these objectives. These solutions produce more volume growth than any of the one and two species mixture alternatives. Although *Pseudotsuga* is only present in the smaller diameter classes, its forced removal (see Table 3) results in a drop in both board and cubic foot growth.

A comparison of Tables 1 and 2 reveals the contrast in diameter distributions when cubic foot volume growth is maximized instead of board foot growth. As expected, fewer large trees are carried in the residual stand when cubic feet are used to measure volume growth. Thus, managerial objectives must be clearly understood prior to optimizing stand management decisions. Also, both solutions nearly extinguish *Pseudotsuga* from the steady-state residual stand. Evident in Tables 1 and 2 is that *Abies* recruitment is limiting the attainment of additional volume production.

Table 1. Three species equilibrium diameter distributions for maximum board foot volume growth.

Board Feet Per Acre Optimal Solution						
Diameter (In.)	Picea	Residual Abies	Pseudotsuga	Picea	Harvest Abies	Pseudotsuga
0- 3	15.89	452.41	13.55	5.86	0.07	61.58
3- 6	4.97	112.19	0.60	0.01	0	3.07
6- 9	1.32	22.87	0	0.14	51.50	1.49
9-12	0.33	4.32	0	2.21	64.10	0.29
12-15	0.08	0.78	0	2.50	34.76	0
15-18	0.02	0.13	0	0.03	0.17	0
18-21	0	0.02	0	0.01	0.03	0
21-24	0	0	0	0	0.01	0
24-27	0	0	0	0	0	0
N_s	22.61	592.72	14.15	MAX $Z_{10} = 10632$ bd.ft./acre		
B_s	2.51	2.14	1.46			
C_s	1.08	1.08	1.60			

The shape parameter (C_s) for all three species shown in both Tables 1 and 2 does not equal one, although it is very close in Table 1 for *Picea* and *Abies*. Thus, with these exceptions, the negative exponential distribution² does not exactly describe the diameter distributions for the three species mixture and, therefore, the distributions are not

²The negative exponential distribution is obtained from the Weibull distribution when C equals one.

balanced. Lastly, 185 and 155 trees per acre must be precommercially removed each cutting cycle to maintain the distributions shown in Tables 1 and 2, respectively.

Table 2. Three species equilibrium diameter distributions for maximum cubic foot volume growth.

Cubic Feet Per Acre Optimal Solution						
Diameter (In.)	Picea	Residual Abies	Pseudotsuga	Picea	Harvest Abies	Pseudotsuga
0- 3	12.65	475.55	0.59	6.81	0.01	72.46
3- 6	0.32	130.31	0.02	2.66	0.01	0.16
6- 9	0.01	5.63	0.01	1.28	77.52	0.06
9-12	0	0.07	0	0.16	64.83	0.01
12-15	0	0	0	0.08	31.58	0
15-18	0	0	0	0	0	0
18-21	0	0	0	0	0	0
21-24	0	0	0	0	0	0
24-27	0	0	0	0	0	0
N_s	12.98	611.56	0.62	MAX Z_{10} = 2220 cu.ft./acre		
B_s	0.70	2.34	0.12			
C_s	0.89	1.64	0.35			

Table 3 illustrates the consequence of forcing Pseudotsuga from the three species solution shown in Table 1. The two species solution shown in Table 3 produces the maximum board foot volume growth of all two species mixtures examined. While not dramatically different from the solution shown in Table 1, the board foot volume growth over the ten-year cutting cycle is 249 board feet less. Further, 295 trees per acre must be precommercially removed each cutting cycle to maintain this distribution. Lastly, the diameter distributions are not balanced. As previously mentioned, the lack of Abies recruitment (and to some extent Picea) appears to be limiting the production of additional board foot volume growth for the two species mixture shown in Table 3. Although not shown, by forcing Pseudotsuga from the three species solution shown in Table 2, the maximum cubic foot volume growth for the resulting Picea-Abies mixture decreases to 2194 cubic feet per acre--a drop of only six cubic feet.

An Abies-Pseudotsuga two species mixture was also examined. The optimal board foot volume production was 10108 board feet per acre each cutting cycle and consisted of 579 and 66 trees per acre for Abies and Pseudotsuga, respectively.

Table 3. Two species equilibrium diameter distributions for maximum board foot volume growth.

Board Feet Per Acre Optimal Solution				
Diameter (In.)	Residual		Harvest	
	Picea	Abies	Picea	Abies
0- 3	22.44	460.94	0.02	0.09
3- 6	3.12	118.70	1.94	0.02
6- 9	0.70	18.61	2.32	58.91
9-12	0.18	2.35	1.41	69.33
12-15	0.05	0.25	1.43	30.86
15-18	0.02	0.02	0.02	0.07
18-21	0	0	0	0.01
21-24	0	0	0	0
24-27	0	0	0	0
N _s	26.51	600.87	MAX Z ₁₀ = 10383 bd.ft./acre	
B _s	1.41	2.19		
C _s	0.83	1.20		

All single species alternatives produced less volume than the two or three species mixtures. However, the single species Abies alternative only underproduced the three species mixture by 691 board feet per acre. Thus, different combinations of species appear to produce approximately equal board foot volumes.

SUMMARY

An optimization program consisting of a deterministic version of the Stand Prognosis Model and a direct search optimization algorithm is used to determine the optimal diameter distribution of each species in an uneven-aged forest stand. Results illustrate the dramatic differences in optimal diameter distributions depending on which measure of volume growth--board or cubic foot--is used in the formulation of the optimization problem. The program also has been used to solve optimization problems with economic objective functions (Bare and Opalach, 1987). Results presented in this paper and in Bare and Opalach (1987) suggest that the optimization techniques employed might be used to optimize any deterministic individual tree/distance independent growth model.

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